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OPEN TOLL LANES IN A CONNECTED VEHICLE ENVIRONMENT

DEVELOPMENT OF NEW PRICING STRATEGIES FOR A HIGHLY DYNAMIC AND DISTRIBUTED SYSTEM

Final Report

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DISCLAIMER

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EXECUTIVE SUMMARY

This project is focused on investigating alternative tolling options in a connected vehicle environment. Future vehicles are expected to have full connectivity and environmental awareness with access to critical system-state information in real-time. Managing toll lanes with congestion pricing is an effective method to address the growing congestion-related traffic problems on freeways. This project focused on developing new frameworks for a tolling system that take advantage of the connected vehicle environment.

This project is closely related to the congestion mitigation focus area of the National Transportation Center @ Maryland (NTC@Maryland) since the proposed research supports the development of effective tolling strategies for congested freeways. In addition, the proposed research will support the Connected Vehicle Initiative of the USDOT since the tolling system to be developed is for a system where vehicles can communicate with the infrastructure.

To investigate future the possibilities for open toll lanes in a connected vehicle environment, the research project was split into two research approaches: analytical and simulation.

The analytical research used a tolling scenario where automobile travelers use V2I (vehicle-to-infrastructure) technology to take part in a Vickrey auction for access to the toll or HOT (high occupancy or toll) lane. Access to the toll road occurs at multiple points along the single facility and the travelers are able to make multiple bids to gain access to all or parts of the toll road. Our research uses this model to explore the impact of varying the distribution of travelers' Value of Time (VOT) on the revenue collected by the toll operator. To this end, three VOT distributions were considered: triangular, log normal, and beta.

Under the Vickrey auction mechanism for the simplified road network, the toll operator will accept approximately one fifth of the drivers onto the toll road of the test network. This result was remarkably consistent across the different VOT distributions and varying road link capacities. The value represents the toll operator's balance between letting more cars onto the toll road and maintaining attractiveness of that toll road. The less attractive the toll road—that is the less the time savings–the less drivers are willing to pay to use it. The results included both intuitive and counterintuitive outcomes. For example, decreasing the capacity of the general purpose road led to an increase in demand for the toll road and more revenue for the toll operator. However, decreasing the capacity of the toll roads had little impact on the results. The results indicate robustness of the auction mechanism design to variation in the distribution of travelers' VOTs.

In addition to the analytical formulations of the problem, a microsimulation test bed was developed to enable assessment of alternative bidding mechanisms for the toll lanes. The simulation was built in VISSIM with custom code. Currently, the model involves a simple network with two parallel routes to evaluate the distribution of traffic between a toll road and a general purpose road. An alternative pricing mechanism based on descending price auction (i.e. Dutch auction) was developed where transactions between drivers and the toll operator are assumed to take place via the V2I technology. Several key components of the simulation test bed have been coded and completed so far as explained in this report. In Phase II of this project, this simulation test bed will be finalized and various tolling scenarios will be designed and implemented to understand the implications of different bidding and tolling mechanisms on the system performance and the toll operator's revenue.

In addition, within phase II of this project, behavioral surveys will be developed and conducted to gain insights into how people would choose to travel on toll roads when given the opportunity to bid. This information will be used to produce an agent-based model (ABM) using the new Agent_Zero framework, proposed by Joshua Epstein (2014), the world's leading ABM researcher, as a means to model human behaviors. This research approach will likely produce more realistic results on tolling in the connected vehicle environment.

1.0 INTRODUCTION

The impetus for fully connected vehicles is strong and growing. Future vehicles are expected to have full connectivity and environmental awareness with all shared critical system-state information in real-time. A large and growing body of research on understanding the safety and mobility implications of connected vehicles already exists. However, there is no significant work on investigating new pricing/tolling options in a connected vehicles world. Congestion or value pricing is an effective method to address the growing congestion problems on freeways. In the current state of practice, toll lanes are typically separated from the regular lanes by physical barriers; toll rates are either fixed or vary by time-of-day or traffic congestion. Vehicles that sense their own locations (including the lanes they are in) and exchange information about their positions and speeds can enable an open tolling system with the number of tolled lanes varying dynamically to maximize the throughput.

Furthermore, the toll operators can directly communicate with each individual vehicle to potentially negotiate toll rates in real-time, similar to auction markets. With autonomous and connected vehicles, it is conceivable to implement such tolling or pricing mechanisms, e.g., those based on auction markets, which allow the drivers to take a more direct role. Compared to the passive role in today's toll roads, drivers would participate in setting a toll rate that is found acceptable by each individual. Under such tolling mechanisms drivers may end up paying varying tolls, consistent with their willingness to pay, for the same service (i.e., trips on the same road segment at the same time of day). Investigating the impacts of such tolling mechanisms on system operations and toll revenue is one of the key goals of this project.

This project focused on developing new frameworks for a tolling system that take advantage of the connected vehicle environment. To evaluate these new tolling systems, various components must be specified and modeled, including specific types of auctions for setting toll rates. The approach taken by the research team to develop these components follows.

1.1 OVERVIEW OF PROJECT

To investigate the future possibilities for open toll lanes in a connected vehicle environment, the research project was split into two research approaches: analytical and simulation. The analytical part of the project focused on the computationally solving a new auction game where drivers, who are the bidders, are able to place bids on using part of the toll road. The simulation part of the project focused on developing a microsimulation environment that allows testing different auction and tolling mechanisms.

The analytical part of the project constructed a simple three-node network with a general purpose and toll road connecting each node. Using a Vickrey auction design (Vickrey, 1961), heterogeneous rational bidders are able to place multiple bids on use of the toll for all or part of their journeys. Determining whose bids were accepted by the profit-maximizing toll operator was the main result from this work. The problem was formulated into a non-linear optimization problem and solved using Wolfram Mathematica (Wolfram Research, 2014). The results explore the impact of using different Value of Time (VOT) distributions for the bidders and the impact of the roadway capacity on the results.

In addition to the analytical formulations of the problem, a microsimulation test bed was developed to enable assessment of alternative bidding mechanisms for the toll lanes. The simulation was built in VISSIM with custom VBA code. Currently, the model includes a simple network with two parallel routes to evaluate the distribution of traffic between a toll road and a general purpose road. Several key components of the simulation test bed are currently coded and completed, as explained in this report. In Phase II of this project, the test bed will be finalized and used to evaluate various scenarios to understand the implications of different bidding and tolling mechanism on the system performance and the toll operator's revenue.

1.1.1 Relevance to the Center Theme

This project is closely related to the congestion mitigation focus area of the NTC theme. The research supports the development of effective tolling strategies for congested freeways. In addition, the research supports the Connected Vehicle Initiative of the USDOT by contributing to innovation in tolling mechanisms relevant to future roadway systems where vehicles can communicate with the infrastructure.

1.2 OVERVIEW OF REPORT

This report is divided into four main sections. The next section describes the analytical research and results. The simulation research documentation follows. The final section discusses the work planned for Phase II of the project.

2.0 ANALYTICAL RESEARCH

Wireless technology increasingly incorporates everyday appliances such as televisions, refrigerators, and even doorbells. Personal vehicles are no exception to this wireless expansion, or "internet of things," with companies like OnStar (www.onstar.com) offering emergency and security Vehicle-to-Infrastructure (V2I) capabilities over cellular phone networks. The evolving nature of V2I technology opens up the possibilities of new applications, such as the participation of travelers in auctions to access toll roads where bids are placed via V2I devices (Zhou and Saigal, 2014). In this analytical research we consider such an application in a simple traffic scenario with dynamic auction tolling. The mathematical model developed in this section shows the impact of varying the distribution of travelers' (customers') Value of Time (VOT) on overall travel time and bid acceptance policies. The results indicate that the proposed auction mechanism is invariant to the VOT distribution, potentially allowing future researchers to reduce their concerns about obtaining an exact VOT distribution for tolling models.

2.1 SECTION OVERVIEW

A brief background on tolling and auctions is given next followed by a detailed description of the theories used in the model. These theories are then framed as an optimization problem to analyze the distribution of travelers' Value of Time (VOT). This section concludes by quantifying abstract variables and solving the optimization problem, giving insight into the use of Vickrey auction schemes for dynamic tolling scenarios.

2.2 BACKGROUND

As volume on transportation networks increases, factors such as environmental constraints, right-of-way issues, societal impacts, and reduced public funding limit the ability and attractiveness of reducing congestion by constructing new roads (Michalaka et al., 2011). When new facility expansion and construction are not options, planners must employ alternative strategies. One popular strategy is the use of High-Occupancy Vehicle (HOV) lanes. Adopting this strategy requires at least two key components, namely the availability of an extra lane for HOV travel and a way to ensure optimal utilization of the dedicated lane. To maximize the potential flow improvements provided by HOV lanes, some locales expand the eligible vehicles to include hybrid electric cars and toll-paying customers. HOV lanes combined with tolls are called HOT (High-Occupancy and Toll) lanes. The first HOT lane was implemented in 1995 on State Route 91 in Orange County, California (Gardener et al., 2013). Thus toll roads could be an effective means to reduce congestion.

Tolling can help relieve congestion while also generating funds for transportation infrastructure improvements and is thus likely to become more prevalent in the coming years. Poole (2014) points out that most of the major interstate corridors were built in the 1960s and 1970s with a fifty year lifespan and are nearing the end of their expected service. He estimates that revitalizing these roads will cost $3.14 trillion over the next 30 years. Tolling might be the only viable means to fund this extensive project, and thus represents an important facet of transportation research.

With the use of non-disruptive technology (e.g. E-ZPass®) which automatically bills the toll road user and eliminates the need for stopping at tollbooths, tolls are becoming easier and more efficient to use. Given the developments in tolling technology and the increasing adoption of this congestion reduction strategy, more research must investigate available tolling policies. Combining the use of mathematical modeling of tolling mechanisms (Wie, 2007, Yang, 2008, Cheng and Ishak, 2013, Zhang et al., 2014) with studies on V2I for traffic management (Milanés et al., 2012), our analytical model examines a scenario where driverless vehicles allow travelers to bid in a live auction for tolls on HOT lanes using V2I technology.

2.3 MODEL THEORY

HOT and normal toll lanes are an increasingly popular solution for congested roadway networks as they give drivers the option to access express lanes. The cost of entry often varies based on demand and no standard method of optimizing these price-points exists. Using the principles of a Vickrey auction that incentivizes "true value" bids, our proposed tolling system utilizes V2I technology to optimize toll operator revenue with toll lane usage. In the scenario, a roadway network consists of a toll lane and a general-purpose lane, each with identical physical properties. Drivers can access the toll lane at the start of the facility or at one interim point along the roadway. Using different statistical distributions to approximate the distribution of travelers' VOT, the model explores the impact of varying the distribution on revenue earned by the toll operator.

2.3.1 Tolling Mechanisms

Two important elements of modeling tolls are the tolling mechanism and the customer utility. The tolling mechanism has traditionally taken the form of a fixed cost for using the toll road; customer utility involves a VOT equation with the customers' choices modeled in a variety of ways (Gardener et al., 2013).

Traditionally, toll roads have fixed prices. One advantage to this approach is that drivers/customers know what to expect and can prepare for the payment amount. As non-disruptive payment methods, such as E-ZPass®, become more common, automated tolling mechanisms reduce the need for tollbooths and introduce the potential for varying toll prices. Dynamic price-points for tolls reflect preset time intervals, such as rush hour, or real-time response to congestion. Examples of real-world dynamic tolls include the San Diego I-15 FasTrak which changes the toll price every 17 minutes during peak periods; the Orange County, California SR-91 tolls which changes every hour; the Minnesota I-394 tolls which changes as frequently as every 3 minutes; and the I-95 express toll lanes in Florida which changes the toll price every 15 minutes (Cheng and Ishak, 2013).

An adjustable tolling mechanism allows control over the number of vehicles using the roadway or toll lane. Setting a toll price too low leads to overutilization, congestion, and service quality degradation. Setting the price too high will discourage use and lead to underutilization of the toll lane and less than optimal congestion relief for the General-purpose (GP) lanes. This research contributes to the theoretical and practical study of tolling mechanisms to ensure an optimal number of vehicles enter the toll lane to both relieve congestion and maximize revenue (Yang, 2008).

Literature on tolling mechanisms reveals a variety of approaches to maximizing revenue. Cheng and Ishak (2013) developed a feedback mechanism for maximizing toll revenue while ensuring that the toll road maintained a minimum speed of 45 mph or higher, which they tested in a VISSIM simulation. Zhang et al. (2008) initially followed a similar approach. Subsequent research by Zhang et al. (2014) adapted this approach to include control theory where a Proportional Integral and Derivative (PID) algorithm was used to control oscillations in flows and make a smoother ride for toll lane users. Wie (2007) applied the Stackelberg leadership model from game theory to solve dynamic toll schedules for a pre-specified subset of arcs with bottlenecks on a congested traffic network. In a broader environment, other influences will cause attraction or aversion to road networks based on tolling prices and practices. Shepard and Balijepali (2012, 2013) modeled the use of toll pricing as a means for cities in competition with one another to attract new residents. Their results indicated that competition resulted in unfavorable tolling strategies for both cities. Friesz et al. (2007) presented a sophisticated method for dynamic congestion pricing; however, they faced computationally intensive and difficult implementation. This problem of computational intensity, known as the curse of dimensionality, plagues many techniques developed to study congestion pricing. The complex nature of the studies described above highlight the difficulty of optimizing vehicle use of the toll road.

Toll price determination usually focuses on the relationship between the customers (travelers) and the toll operators. With V2I technology, the potential exists for the toll operator to directly interface with the travelers in real-time in response to road conditions. The toll operator would respond to higher or lower demand by raising or lowering the price accordingly. During times of high-volume, however, there is little room for error in determining the toll price. Setting the price too low could lead to congestion on the toll road beyond that of the GP lanes, leading to customer dissatisfaction. Rather than allowing the toll operator to randomly allocate eligible access to the toll road when demand is higher than capacity, we propose the use of an auction to determine which vehicles will pay what price to enter the toll lane.

The following auction mechanism is proposed to dynamically adjust toll amounts based on travelers' demonstrated valuation of road use provided via V2I technology by placing bids for the maximum amount they are willing to pay to use the toll road, an amount roughly equivalent to their individual value of being admitted to the road, without exceeding desired density levels. This version of the auction mechanism is a *one-shot* auction, but there are many different ways to run such an auction (for an overview of standard auction types, see Teodorovic et al. (2008) and the study of Mechanism Design by Dash et al. (2003)). Different auction mechanisms will result in different revenue payouts. The toll operator then determines a cut-off price for bids based on the number of vehicles needed to reach desired density levels, so tolling mechanisms should adapt to suit the needs of the specific scenario.

Our research incorporates a Vickrey auction mechanism. A Vickrey auction works by allowing the auctioneer to set a cut-off for the number of winning bids and then accepting that number of bids. The final price given to the "winners" is equal to the highest bid that was *not* accepted (or slightly above it) (Vickrey, 1961). The proxy bidding system of the eBay® website is similar to a Vickrey auction. The advantage of this auction mechanism is that it disincentivizes out-bidding behavior where auction participants attempt to "game" the system. Instead, consumers tend to bid their perceived values of the product, leading to realistic valuation of the auction item, in this case toll prices. Our mechanism design does allow for multiple bids from travelers. This creates some challenges discussed later in this section.

Though the Nobel Prize winning economist William Vickrey published research on both congestion pricing problems (Vickrey, 1969) and auction theory (Vickrey, 1961) in the 1960s, he did not connect the two together at the time. This is likely due to the infeasibility of bidding while driving and lack of means to organize bid outcomes in real-time, problems that modern technology solve. With the potential use of V2I technology and driverless vehicles, it is now feasible to conduct a tolling auction during transit. Until now, research connecting Vickrey's two ground-breaking ideas has focused on bidding before travel rather than bidding while travelling. For example, Teodorovic et al. (2008) proposed an auction-based congestion pricing scheme for people to bid on entry to a downtown area in a week period, where the bidders wish to make one or more visits. The researchers produced a mixed integer program problem and solved it using heuristics. The study did not consider V2I technology.

One recent paper that considered V2I-facilitated auctions was Zhou and Saigal (2014) who used a combinatorial auction approach to process bids from an interconnected toll road network. Combinatorial auctions allow bidders to bid on different, or even multiple but overlapping items such as an interconnected toll network (Cramton et al., 2006). The mechanism that Zhou and Saigal used is called the VCG named after Vickrey (Vickrey, 1961), Clarke (Clarke, 1971) and Groves (Groves, 1973). The VCG works by first deciding which bids maximize revenue and then determining the bid price. This price is based on the difference between the revenue gained if the bidder's bid was accepted and the theoretical revenue gained if the bid was not accepted (note that if a bid was not accepted then other bids might be accepted instead). The VCG approach is very computationally intensive and is NP-hard (Teodorović et al., 2008).

In this research, we connect Vickrey's auction mechanism with V2I technology to optimize revenue and lane usage without requiring excessive computation. This first requires quantifying the driving behavior factors considered to evaluate efficiency and customer satisfaction. The following section establishes the customer utility aspect of this type of tolling mechanism.

2.3.2 Customer's Evaluation of Time

The majority of tolling research, including this study, uses a concept called Value of Time (VOT) for determining how much travelers are willing to pay. VOT is a linear multiplying constant that relates time saved to a monetary value. The authors accept that using VOT has several drawbacks, including that it does not take into account the complex factors that go into human decision-making. For example, travelers' VOT reflects time of day as indicated by the VOT for the I-395 HOT lane in Minnesota that varied from $73/hr in the morning and $116/hr in the afternoon (Cho et al., 2011). Drivers also exhibit a perception bias toward tolls of about 15-20 minutes, where they will accept an increase in travel time to avoid a toll (Transportation Research Board, 2013). Additionally, the research suggests that travelers value time savings more on longer trips (Transportation Research Board, 2013) and they are only willing to accept time-varying toll charges if it can yield more reliable journey times (Bonsall et al., 2007). These factors could explain why Cho et al. (2011) discovered a limited correlation between time saved and proportion of travelers using HOT lanes. These human factors indicate that VOT may require the addition of a constant value. Mickalaka et al. (2011) present more complex examinations of VOT using artificial intelligence techniques to learn travelers' dynamic VOT using data from the I-95 Express in Florida, but this is beyond the scope of the current study.

In this model, the VOT takes a back seat to the auction pricing mechanism, our primary area of inquiry. For that reason, the inability of VOT to capture more complex human decision-making processes is outweighed by the simplicity of the approach. In this study, the heterogeneity of VOT is the most important aspect of this measure, rather than the actual function.

A final aspect of consumer behavior incorporated into the model involves determining how drivers make the choice to participate in dynamic tolling auctions. In the literature on toll participation, the favored approach is to use logit choice models (Talluri and van Ryzin, 2004) to replicate human decision-making about whether to take the toll road or not (Cheng and Ishak, 2013, Zhang et al., 2008). Gardener et al. (2013) did a comparative study of customer choice models and concluded that the Burr distribution was the best fit for customer choice. In our model, we assume that every traveler has a value for the time saving aspect of using the toll road (even if that value is zero), so every vehicle places a bid. This means that a traveler's only choice in our model is the amount of his or her bid, which we have assumed is based on that traveler's VOT.

2.3.3 Model Design

The research presented in this analytical section considers a simple tolling scenario (Figure 1) to investigate the impact of assuming different estimates of the VOT statistical distribution on both the toll operator's revenue and the travel times of the vehicles in the system.

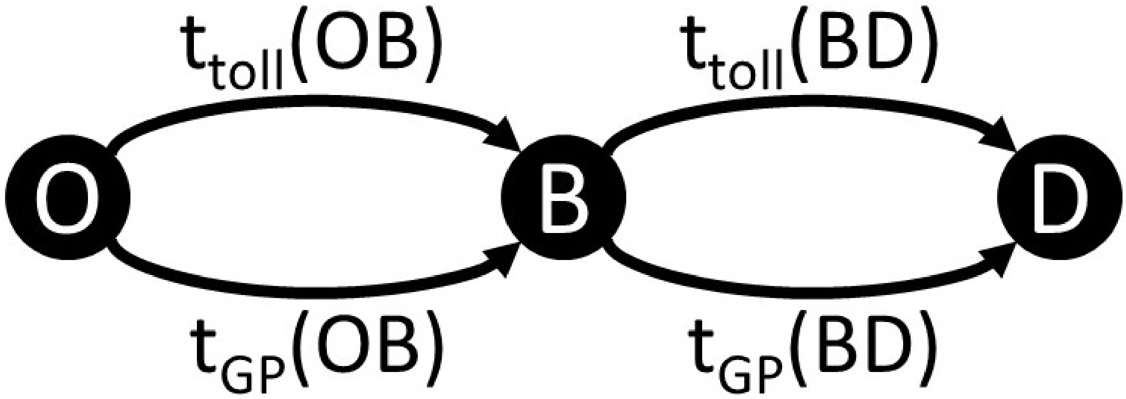

Figure 1: Node-arc diagram of scenario transportation network.

The scenario involves a single origin-destination pair, *OD*, connected by a general-purpose (GP) road and a toll road. At the midway point, between nodes *O* and *D*, there is an interchange *B* between the toll road and the GP road (Figure 1). We assume that movement between the two roads at the intersection is smooth and does not interfere with the flow of traffic. All segments are identical in length and capacity. Thus, given the same vehicle volume, travel time is the same on all road segments. The total volume of vehicles is normalized to equal one, without loss of generality.

Unlike traditional fixed tolling mechanisms, travelers in this scenario bid to use the toll road and the toll operator determines which bids are accepted. The travelers initially place two bids: one for use of the toll road from *O* to *D* without interruption (b(OD)) and one for the toll road from *O* to *B* (b(OB)). If a traveler's bid b(OD) is accepted, his other bids are nullified. Travelers whose initial b(OD) bids are not accepted are provided the opportunity to bid for the BD segment thus there are three potential bidding opportunities in total. Notice that a traveler may get to use the toll road for the complete journey even if his b(OD) is rejected as his b(OB) and b(BD) bids might be accepted.

Toll or HOT lane operation policies may have multiple objectives. The most common are to maximize throughput of the entire freeway (both GP and toll lanes) and to provide free-flow traffic service on the toll lane. We assume in this scenario that the toll operator's only objective is to maximize revenue from the tolls collected. Thus the operator is not concerned with minimizing overall travel time or maximizing road efficiency.

In this scenario, we assume that all vehicles and toll operators have the V2I technology required for this process to occur. Additionally, we assume all vehicles can place bids before reaching the selection nodes *O* and *B*. We also assume that there is no slowdown from a vehicle entering or exiting a toll lane. Finally, we assume that all travelers are able to honor their bids. There are three parts required to produce a complete toll model: the link travel time function, the bidding mechanic, and the customer's VOT.

2.3.4 Link Travel Time Function

As VOT is directly related to bids placed by travelers, the time required to transverse the network and the effects of congestion are critical. The Bureau of Public Roads (BPR) developed a standard equation for congestion on road segments, shown in Equation (1), the use of which is also supported by Teodorović et al. (2008). This equation is based on Greenshield's (1935) "fundamental diagram of traffic flow."

$$t(l,v) = t_{ff}(l)\left(1 + 0.15\left(\frac{v}{c(l)}\right)^4\right) \tag{1}$$

This equation determines the travel time t of a link l for a traffic volume v for a given free-flow travel time t_{ff}, and the road segment capacity c. Justified by its acceptance in congestion and tolling studies, this equation will also inform the model development for our study.

2.3.5 Bidding Mechanism

Since the entire bidding process in this scenario occurs over wireless V2I technology, there are two decision-makers in the system, the travelers and the toll operator. The equations below represent the decisions of the travelers. The decision of the toll operator is discussed later. Based on individual VOT, the travelers will place bids for access to toll road segments. We assume that travelers have perfect knowledge about travel time on the road segments and are thus able to determine the travel time savings of using the toll road. The justification for this perfect knowledge is the assumption that regularly commuting travelers along the road would likely be able to make accurate estimates of travel time based on the current conditions. By using a Vickrey auction mechanism, the travelers lack incentive to bid anything other than their true estimates of the toll price. Based on these assumptions, a bidding formula for the travelers for arc OB is given below:

$$b(OB,x) = u(x)\big(t\big(OB_{GP}, v(OB_{GP})\big) - t(OB_{toll}, v(OB_{toll}))\big) \qquad (2)$$

A bid b of traveler x is determined by multiplying their value of time u by the travel time savings between the general-purpose and toll lanes. The advantage of using this bidding equation is that it stops the scenario where the toll operator just accepts all bids. This situation would make the toll road's congestion worse than the GP lane, leading the travelers to bid zero.

$$b(OD,x) = b(OB,x) + b(BD,x) \qquad (3)$$

A variation of equation (2) can also be used for bids of the *BD* road segment. Determining bids for using the toll road all the way from *O* to *D* (b(OD)) is trickier because it involves multiple road segments (equation (3)). We assume that this bid only considers travel time savings, which means that it relates to equation (2). This estimate of the OD bid raises some concerns, which we discuss in the results section.

2.3.6 Travelers' VOT Distribution

The model assumes that travelers propose heterogeneous bids; otherwise everyone would bid the same amount. Here we establish a method for modeling travelers' VOT. This means that each traveler will have a different VOT defined as u(x) in the equations given above. Thus there is a need to define the distribution of the traveler's VOT. Previous studies have used different fare classes for determining VOT (Yang et al., 2002, Han and Yang, 2008); however, it is has recently be advocated that VOT should follow a continuous probability distribution (Liang and Mahmassani, 2013). Three different distributions are considered as candidates for our model: triangular, log normal and beta.

Without any estimate of the distribution of VOT, we lean towards parsimony and initially apply a simple triangular distribution. The triangular distribution allows us to set a minimum VOT, i.e., zero, and a maximum VOT (even billionaires have limits on their values of time). Unlike the uniform distribution, the triangular distribution also allows us to set an average: the mode. We can then vary this mode to produce different distributions (Figure 2). Other functions, like the Beta function, achieve this variability but require more complex implementation. The triangular function is used in other transportation models as an approximation of relationships like flow to density (Michalaka et al., 2011). Since the function is continuous, we assume that the number of vehicles is continuous and not discrete. To simplify the mathematics, we normalize the total number of vehicles to one.

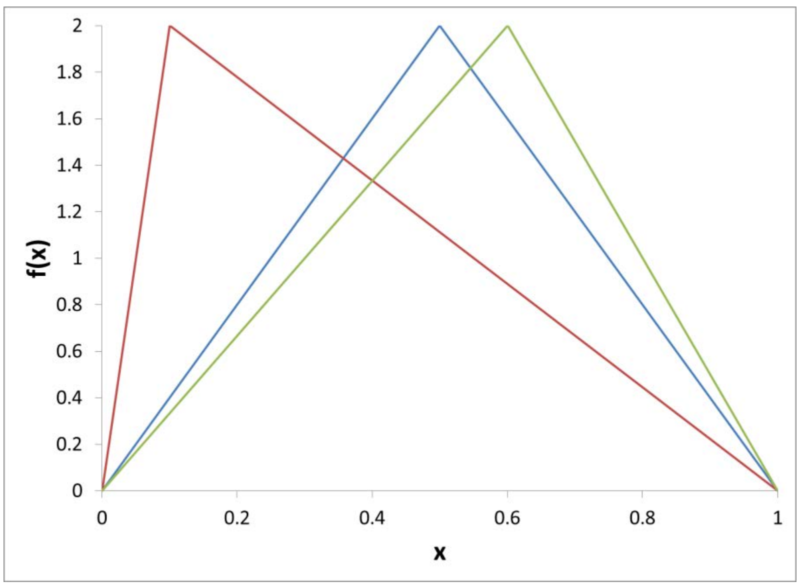

Figure 2: Various versions of the triangular distribution with a minimum of zero and a maximum of one.

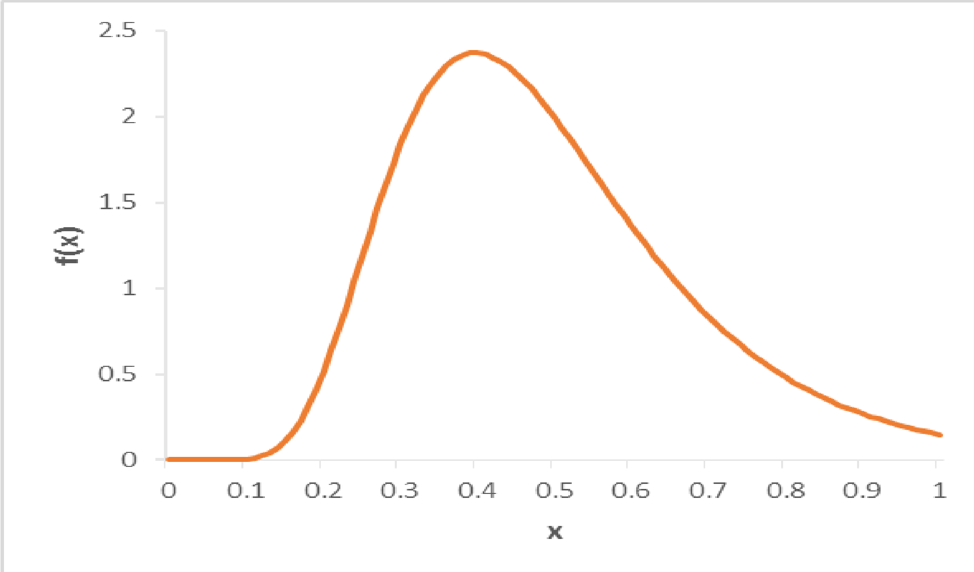

Figure 3: An example of the log normal distribution

However, triangular distribution might not be the most appropriate distribution for VOT. Two other distributions are also considered in this work, namely the log normal and the beta distribution. The log normal distribution can have a peak like the triangular distribution but is also smooth (Figure 3). However, the log normal does not have an upper limit to its range. The beta distribution (Figure 4), used in Bayesian statistics, is also smooth but with an upper limit (of one). However, the beta distribution can be difficult to analytically manipulate, as discussed later in this section.

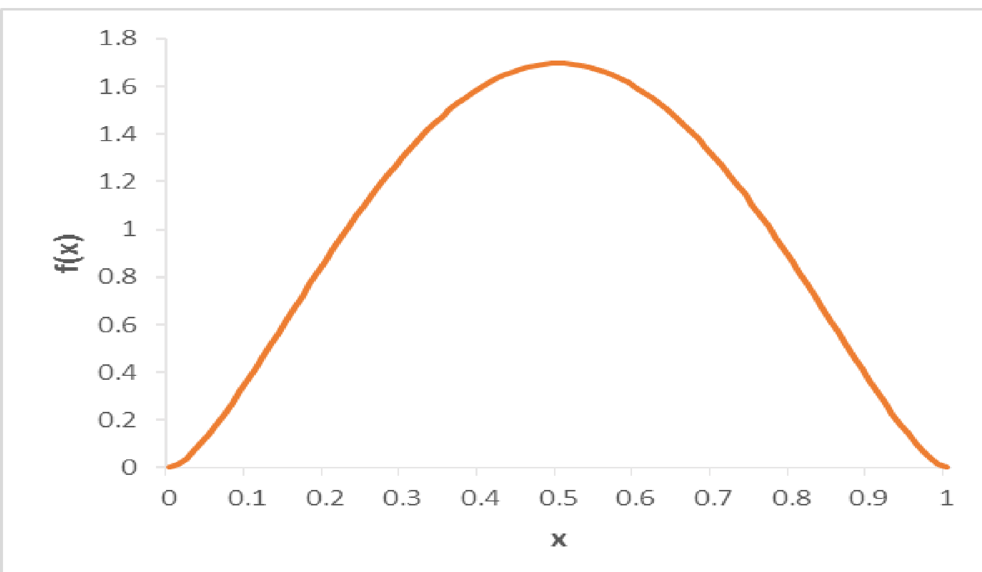

Figure 4: An example of the Beta distribution

2.3.7 Optimization Problem

Here we develop the model that helps the second set of decision-makers, the toll operators, to decide which bids to accept given the travelers' bidding strategies discussed above. Due to the structure of Vickrey auctions, the accepted price for toll road access is the value of the lowest accepted bid, or highest non-accepted bid in the continuous case. The toll operator then faces a dilemma: as more vehicles are accepted, the number of payments collected increases, but the toll price for each is lower. The toll operator must balance the number of vehicles accepted against the actual toll price paid. Also, the more vehicles accepted onto the toll road, the less attractive it is to drivers (more vehicles mean slower travel times), which will lower their initial bids. The toll operator must consider these factors to maximize toll revenue. There are three decisions that the toll operator must make:

- The number of bids to accept for use of the entire length of the toll road (b(OD))
- The number to accept for just the first toll segment only (b(OB))
- The number to accept for last toll segment only (b(BD))

We make the assumption that the toll operator takes the highest bids available for each segment. We also make the assumption that the operator prefers bids for b(OD) over b(OB), as more guaranteed revenue is generated. Based on these assumptions, we develop the following optimization problem:

$$\begin{aligned} \max_{\theta,\mu,\lambda\in[0,1]} \int_{F^{-1}(1-\lambda)}^{u_{max}} & b\big(OD, F^{-1}(1-\lambda)\big) f(x)dx \\ & + \int_{F^{-1}(1-(\theta+\lambda))}^{F^{-1}(1-\lambda)} b\left(OB, F^{-1}\big(1-(\theta+\lambda)\big)\right) f(x)dx \\ & + \int_{F^{-1}(1-(\mu+\lambda))}^{F^{-1}(1-\lambda)} b\left(BD, F^{-1}\big(1-(\mu+\lambda)\big)\right) f(x)dx \end{aligned} \tag{4}$$

Such that

$$\theta + \lambda \leq 1$$
$$\mu + \lambda \leq 1$$

The proportion of travelers that have their b(OD) accepted is λ, the b(OB) accepted is θ, and the b(BD) accepted is μ. The functions F and f are the cumulative distribution function (CDF) and probability distribution function (PDF) of the VOT distribution respectively. We identify the vehicles by their associated VOT, thus u(x) = x here; since the vehicles are represented as continuous variables their VOTs are unique. The constraints ensure that no more vehicles are allocated to the arc OB_{toll} and BD_{toll} than there are vehicles in the system (which we normalized to one). The three integrals show the total revenue generated from the three groups: those accepted for travel on the complete toll road and those accepted for travel on only one of the two segments. Notice that the price paid by each vehicle in a particular group is constant and is equal to the lowest bid offered for that group. Thus the equation can be simplified to:

$$\max_{\theta,\mu,\lambda\in[0,1]} \lambda b\big(OD, F^{-1}(1-\lambda)\big) + \theta b(OB, F^{-1}(1-(\theta+\lambda))) + \mu b(BD, F^{-1}(1-(\mu+\lambda))) \tag{5}$$

Before manipulating the optimization equation further, we make some simplifying assumptions. We have assumed that all the arcs are identical length and we normalize the free flow travel to get $T_{ff}(\cdot) = 1$. Volume of traffic is required for the travel time equation which we assume is just the proportion of vehicles that use that particular arc. This means that we are assuming discrete traffic flow while assuming that the number of vehicles is continuous. Based on these assumptions equation (5) becomes:

$$\begin{aligned}\max_{\theta,\mu,\lambda\in[0,1]} & 0.15\left(\lambda F^{-1}(1-\lambda)+\theta F^{-1}\big(1-(\theta+\lambda)\big)\right)\left(\left(\left(\frac{1-(\theta+\lambda)}{c_{GP}(OB)}\right)^4 - \left(\frac{(\theta+\lambda)}{c_{toll}(OB)}\right)^4\right)\right) \\ & + 0.15\left(\lambda F^{-1}(1-\lambda)+\mu F^{-1}\big(1-(\mu+\lambda)\big)\right)\left(\left(\left(\frac{1-(\mu+\lambda)}{c_{GP}(BD)}\right)^4 - \left(\frac{(\mu+\lambda)}{c_{toll}(BD)}\right)^4\right)\right)\end{aligned} \tag{6}$$

s.t.

$$\theta+\lambda \in [0,1]$$
$$\mu+\lambda \in [0,1]$$

The capacity of the road segments is given by $c_{gp}(\cdot)$ for the general purpose segments and $c_{toll}(\cdot)$ for the toll segment. The capacity of each segment is varied for the results section to gain an understanding of its effects on the selection of toll users by the toll operator. Equation (6) reduces the number of additive parts of the equation (5) from three to two; this happens because of the relation of the bids given in equation (3). At this stage, equation (6) could use any probability distribution for the travelers' VOT.

2.3.8 Triangular Distribution VOT case

The first VOT distribution we considered is the triangular one. This distribution is incorporated into the optimization problem by substituting its inverse CDF into equation (6). The triangular distribution's CDF is a continuous function, but involves different functional parts for each side of the mode value. To keep our equations simple, we are going to assume the following:

$$\begin{aligned} F^{-1}\big(1-(\theta+\lambda)\big) &\geq u_{mod} \\ F^{-1}\big(1-(\mu+\lambda)\big) &\geq u_{mod} \end{aligned} \tag{7}$$

u_{max} is the upper bound of the triangular distribution and u_{mod} is the distribution's mode. Under these constraints, we only need to consider one side of the triangular distribution, thus:

$$F^{-1}(1-x) = u_{max} - \sqrt{x.u_{max}(u_{max} - u_{mod})} \tag{8}$$

This means that the new constraints become:

$$\begin{aligned}(\theta+\lambda) &\leq \frac{u_{max} - u_{mod}}{u_{max}} \\ (\mu+\lambda) &\leq \frac{u_{max} - u_{mod}}{u_{max}}\end{aligned} \tag{9}$$

Substituting equation (8) into equation (6) results in the follow optimization problem, with the constraints above, and setting all the capacities to one:

$$\begin{aligned}\max_{\theta,\mu,\lambda\in[0,1]} 0.15\Bigg(&(\theta+\lambda)u_{\max} \\ &- \sqrt{u_{\max}(u_{max}-u_{mod})}\Big(\lambda\sqrt{1-\lambda} \\ &+ \theta\sqrt{1-(\theta+\lambda)}\Big)\Bigg)\left(\left(\left(\frac{1-(\theta+\lambda)}{c_{GP}(OB)}\right)^4 - \left(\frac{(\theta+\lambda)}{c_{toll}(OB)}\right)^4\right)\right) \\ &+ 0.15\Bigg((\mu+\lambda)u_{max} \\ &- \sqrt{u_{\max}(u_{max}-u_{mod})}\Big(\lambda\sqrt{1-\lambda} \\ &+ \mu\sqrt{1-(\mu+\lambda)}\Big)\Bigg)\left(\left(\left(\frac{1-(\mu+\lambda)}{c_{GP}(BD)}\right)^4 - \left(\frac{(\mu+\lambda)}{c_{toll}(BD)}\right)^4\right)\right)\end{aligned} \tag{10}$$

This equation is used to determine the bids accepted and return gained for various different input parameter values, which is discussed in the results section. In this report, we only focus on the triangular distribution when the maximum value is one and a mode of 0.5. For details on the effects from varying the mode, please see Collins et al. (2015).

2.3.9 Log Normal Distribution VOT case

The next VOT distribution to consider is the log normal distribution. The equations associated with the log normal are more complex than the triangular distribution, as seen by its CDF function:

$$F(x;\mu,\sigma) = \frac{1}{2}\left(1 - erf\left(-\frac{ln(x)-\mu}{\sigma\sqrt{2}}\right)\right) \tag{11}$$

The log normal distribution uses the error function which is defined as:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{12}$$

Using Equation (11), it is possible to determine the inverse CDF function:

$$F^{-1}(1-y; \mu, \sigma) = e^{\mu - \sigma\sqrt{2} erf^{-1}(2y-1)} \tag{13}$$

Thus substituting equation (13) into equation (6) results in the following optimization problem:

$$\begin{aligned} \max_{\theta,\mu,\lambda \in [0,1]} 0.15\Big(& \lambda e^{\mu - \sigma\sqrt{2} erf^{-1}(2\lambda - 1)} \\ &+ \theta e^{\mu - \sigma\sqrt{2} erf^{-1}(2(\theta+\lambda)-1)}\Big)\left(\left(\left(\frac{1-(\theta+\lambda)}{c_{GP}(OB)}\right)^4 - \left(\frac{(\theta+\lambda)}{c_{toll}(OB)}\right)^4\right)\right) \\ &+ 0.15\Big(\lambda e^{\mu - \sigma\sqrt{2} erf^{-1}(2\lambda - 1)} \\ &+ \mu e^{\mu - \sigma\sqrt{2} erf^{-1}(2(\mu+\lambda)-1)}\Big)\left(\left(\left(\frac{1-(\mu+\lambda)}{c_{GP}(BD)}\right)^4 - \left(\frac{(\mu+\lambda)}{c_{toll}(BD)}\right)^4\right)\right) \end{aligned} \tag{14}$$

With the same constraints as given in equation (6). One can observe that the inverse error function has not been explicitly defined here; that is because there exists no simple analytical form for the inverse error function. Even the McLaurin series for the function requires multiple functions to be defined. Thus, though the log normal distribution maybe smooth, it does not produce as an elegant a formulization of the problem as the triangular distribution.

2.3.10 Beta Distribution VOT case

The beta distribution provides not only a smooth distribution but also a bounded one. However, like the log normal distribution, it analytical manipulation is far from elegant. Though the CDF function can be analytically defined, as seen in equation (15), its inverse function cannot. The CDF for the beta distribution is given by the regularized incomplete beta function:

$$I_x(\alpha, \beta) = \frac{B(x;\ \alpha, \beta)}{B(\alpha, \beta)} = \frac{\int_0^x t^{\alpha-1}(1-t)^{\beta-1} dt}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt} \tag{15}$$

$$x \in [0,1]$$

Given this lack of definition for the inverse CDF, we rely solely on the numerical appropriation, provided in Mathematica 10, for enumerating this function.

2.4 RESULTS

The numerical solutions for the three different optimization problems were solved using Wolfram Mathematica 10 (www.wolfram.com/mathematica/) and the results are summarized in Appendix

A. The results given in each of the tables shows the percentage of the overall population of bids accepted for each bidding possibilities: OD, OB, and BD. The table also shows the expected gains for the toll operator, which is discussed further below. The different rows of the tables reflect the different capacity cases for the each of the road segments. Only two roadway capacity options were considered: one or one half. The combination of this simple binary variation lead to some interesting results across the three different distributions.

To aid with the comparison between VOT distributions, parameters were chosen to ensure that all three distributions had the same mean and variance of 0.5 and 1/24. For the uniform distribution this meant setting the mode to 0.5 and the maximum to 1. For the log normal distribution, this meant the underlying normal distribution had a mean of -0.7702 and a standard deviation of 0.39262. For the beta distribution, this meant that the alpha and beta values were set to 2.5. In all three cases, determining the underlying parameter values to ensure the correct mean and variance was achieved by solving simultaneous equations relating the associated formulas for each distribution. We have omitted this documentation from this report in the interest of space.

Given the heavily non-linear aspects of the three optimization problems, they were numerically solved using the *NMaximize* function in Mathematica 10, which was set at a precision of five significant digits of accuracy. The *NMaximize* function runs various heuristic methods to find the solution to the level of accuracy required. These methods include Simulated Annealing (Van Laarhoven and Aarts, 1987) and Nelder-Mead method (Nelder and Mead, 1965). Thus we can only conclude that our solutions are correct for a level of accuracy of five significant digits, and the actual solutions might result in different first five significant digits (though we highly doubt this is the case).

There are two types of results found in the numerical solutions: acceptance percentage and return. The acceptance percentage is simply the percentage of the bidding vehicles whose bids are accepted. Thus if 7% of bids are accepted of the OD bids and 10% of the OB, then the OB toll road will have 17% of vehicles admitted. The return is a little more difficult to understand as it is based off of the amount of money the toll operator would get from a continuous number of vehicles. Thus it is recommended that the reader consider returns in the relative sense only, i.e., for comparing to other returns produced from other scenarios. A simple example gives the reader some empirical understanding of the return value: consider the case when each segment has then same capacity (1000 vehicles per hour) and free-flow speed (one hour to transverse) and the average value of time is $20 an hour; then if the volume is approximately the capacity, the toll operator would expect to generate $362 per hour with prices of $2 per vehicles (based on the result of the first scenario return of 0.0181 for the triangular distribution).

There are many differences between the three sets of results but there are also many similarities. It is these similarities that will be discussed first as they give some deeper understanding of the overall system. First of all, the case when the general purpose segments are reduced to half capacity (and the toll road remains the same) produces the highest return for the toll operator for all three distribution cases. This is unsurprising as this is the worst situation for the GP segments of all the scenarios considered, thus the drivers will experience the greatest time savings from using the toll road. When the toll road's capacity is also reduce by half, we notice only a slight reduction in this return for the maximum case. This slight reduction is due to the toll road only be slightly less favorable because of the general underutilization of all the toll segments. Most results only indicate the toll road being used at 17% capacity (or 34% in the case of the reduced capacity toll segment). This underutilization is not surprising because the toll operator will want to keep the traffic flowing as fast as possible on the toll segments to ensure the largest travel-time difference between the toll and GP segments which, in turn, increases the price that drivers are willing to pay to use the toll segment. If the toll operator increases the number of cars on the toll segment, this will increase the travel time of the toll road and thus reduce the amount any driver is willing to pay to use it.

Another result that is consistent across the three VOT distribution cases is a mirroring of the results between the OB and BD. That is, if the capacity values for the OB segments are swapped with the BD segments then the acceptance percentages for OB and BD are also swapped. The OD percentages and return stay the same.

Though the returns are different, there seems to be a consistency of the acceptance percentages across all scenarios and parameter values. Approximately 16-17% of drivers are accepted onto the toll segments (both OB and BD) in all cases

2.4.1 Comparison of Results

The relative comparisons of results from varying the parameters are not consistent across the three VOT distribution scenarios. For example, the acceptance percentages found for the case when all the segments have full capacity are the same as when the segments have half capacity. This is true for the log normal and beta distribution cases but not for the triangular distribution case. Thus general conclusions about the impact of road capacity cannot be drawn for all three scenarios. Some conclusions about return can be made, e.g., decreasing GP capacity increases return and decreasing toll capacity decreases return.

The difference between the three distributions' returns is strikingly similar, as shown in table A-4, which can be found in Appendix A. The log normal distribution results are only 1-2% less than the triangular distribution's returns. This slight decrease is probably due to the longer tail of the log normal distribution, thus more customers are willing to pay a higher price for use of the tolls. The beta distributions results are only a fraction of a percentage greater than the triangular distribution's results. This slight increase is probably due to the beta distribution having a higher Kurtosis (peak) than the triangular distribution (thus less in the tails).

2.5 DISCUSSION ON APPLICATION

Results from the model indicate that when the toll operator maximizes its revenue, the percentage of auction bids accepted for toll road access is robust to changes in the VOT distribution. This equates to approximately 17% of vehicles accessing the facility. Considering the difficulty in obtaining actual travelers' distribution of VOT, our auction tolling mechanism implies that obtaining an exact VOT distribution may not be necessary for this type of tolling analysis as three different distributions were considered here.

The discussion so far has been academic in nature and, as such, has not considered the practical limitations of implementing such a toll auction mechanism. Four practical limitations come to mind: education, participation rates, perception, and the dead fish fallacy.

Existing variable tolling mechanisms may be more intuitive than the one proposed here. Though the proposed bidding mechanism is simpler than other auction types, it is still a combinatorial auction and its mechanics might be beyond the grasp of the general population. From a practical perspective, the traveler must understand that multiple bids happen simultaneously and correspond only to certain segment(s) of the roadway network. Additionally, placing a bid does not guarantee acceptance to the toll road as only the selected top bidders gain access. In addition to the ubiquitous spread of V2I technology that would be required to implement our proposed tolling mechanism, the toll operator would likely need to pursue an extensive advertising and public relations campaign before deployment.

The current model assumes 100% participation rates, which is highly unlikely to occur amongst the general population, especially for those with a low value of time. In practice, the V2I technology and tolling mechanism would need a component to allow for default bid values. These would allow travelers to set a very minimal bid that they are always willing to pay for use of the toll road. The current optimization model does not account for this option. The impact of default bids on an auction-based tolling mechanism is an area for further study.

An auction-based system for HOT lanes could result in negative public perception. Ethical considerations for who has access to the toll or which bids are accepted constitute a hurdle for HOT implementation in general. Given that accepted bids reflect the highest unsuccessful bid, uninformed travelers may be skeptical of the method for selecting winning bids as prices constantly vary.

The final concern we discuss here is that our system assumes steady state because the drivers in the scenario have perfect knowledge and have a constant VOT that is not affected by the system. We specifically made these assumptions to explore the mathematical theorizing of a Vickrey auction tolling system, but they do not reflect real-world human dynamics that are constantly in flux. Salt (2008) calls this the Dead Fish Fallacy , which highlights the idea that despite the non-static, dynamic nature of the world, scientists insist on assuming static behavior. This has consequences on the experimental and/or observational results drawn. To illustrate this, Salt colorfully draws a parallel by noting that the only time fish remain static in a pond is when they are all dead. In the case of a Vickrey auction tolling mechanism, we submit to the Dead Fish Fallacy in order to assume that a steady state is an approximation for the long-term behavior of the tolling system, but not for the initial implementation term. Finally, we assume that travelers' VOTs are not affected by the system which maybe an unreasonable in the real-world. For example, if you are willing to pay $10 to use a toll road but the final price consistently comes back at $1 then might you question why your VOT is so high? The market itself, and our own social networks, influence our VOT and may bring the overall revenue potential down (Seiler et al., 2013). As technology advances to make the proposed tolling mechanism more feasible, more research is required to address these limitations and account for human heterogeneity of VOT and decision criteria to ensure optimal revenue and lane usage.

2.6 ANALYTICAL RESEARCH CONCLUSIONS

This section described and modeled a tolling scenario where automotive travelers use V2I technology to take part in a Vickrey auction for access to the toll/HOT lane. Access to the toll road occurs at multiple points along the single facility and the travelers are able to make multiple bids to gain access to all or parts of the toll road. Our research used this model to explore the impact of varying the distribution of travelers' VOT on the revenue achieved by the toll operator. To this end, three VOT distributions were considered: triangular, log normal, and beta. The results indicate robustness of the auction mechanism design to variation in the distribution of travelers' VOTs. This affected the revenue obtained by the toll operator and the amount of vehicles accepted on the toll road, which was about 17% of the total vehicles entering the system.

The advantage of using V2I tolling is that the toll operator can dynamically adjust the toll price to ensure full utilization of the toll road. In our model, however, road utilization maximizes the toll operators' revenue without attempting to necessarily provide the best option for the travelers. The socially optimal solution, in terms of average travel time, would be to allow equal numbers of cars on the toll lane as the general-purpose one. This would remove the benefit of using the toll lane over the GP lane, and would thus disincentivize bidding and reduce the toll operator's revenue to zero. There is an incentive, then, for the toll operator to keep the GP lane congested, which follows the old business adage of creating a demand for one's product. Since the travelers bid on use of the toll road, it also means that their satisfaction with toll operator's acceptance strategy is implicitly included in this bid. This system keeps the toll operator interested only in the paying travelers. Those who win the Vickrey auction are the ones with the highest Value of Time, i.e. the richest. This system therefore leads to some embedded socioeconomic inequalities that surface through the use of this tolling strategy.

If our system is any reflection of reality, then its invariance to the population's distribution of VOT indicates that auction-based tolling could be an unnecessary mechanism because the population makeup and preferences have very little effect on the outcome. It is possible that traditional dynamic tolling mechanisms perform at least as well as the Vickrey auction version, but further research is required to determine if the model results presented here truly reflect reality.

3.0 SIMULATION RESEARCH

In this section, a new pricing methodology is investigated based on descending price (or Dutch) auction in a connected vehicle environment. Also, an agent-based simulation model for testing and analysis of the proposed algorithm is built. For simulation model building, development, and scenario analysis, VISSIM microscopic traffic simulation software is used with an external Visual Basic for Applications (VBA) script implementing the proposed methodology and the behavior of agents. Once completed in phase II of this project, the simulation model will be used to compare the previously introduced analytical model of Vickrey auction to a new auction strategy and assess its effects on revenue collection and throughput on toll lanes.

3.1 SECTION OVERVIEW

A brief description of the problem and literature review on descending price (Dutch) auction and its use in other relevant markets follows. The literature review is followed by the details of proposed agent-based modeling and simulation (ABMS) and its development stages. The brief explanation about how the current algorithm was developed and will be analyzed concludes the simulation research section.

3.2 LITERATURE REVIEW

3.2.1 Descending Price (Dutch) Auction

An auction is a competitive bidding process where the parties negotiate over the prices through different mechanisms. Throughout the negotiation process, the auctioneers' main aim is to sell the item or items to their clients at the highest possible price at which both parties agree. Auctioning is used as a very successful model today in numerous business sectors (Li and Kuo, 2011).

Dutch auctions, as it can be easily predicted from its name, derives from the traditional tulip markets of the Netherlands and they are often known as clock auctions or one-sided auctions (Li and Kuo, 2011). Unlike the ascending price in English auctions, in this type of auction, the selling value of the item starts decreasing from an arbitrary price, which is high enough to outbid all offers that might come from the clients. The auction price falls according to a predetermined price decrement rate at regular time intervals and the price decrement stops once it reaches a reserve price predetermined by the auctioneer. Each price decrement is considered as a different round. Therefore, this type of auction is also a type of multiple rounds auction. Moreover, the auction clock ends once a client calls out the acceptance of the price (NYSERDA, 2004).

While the ascending price (English) auction is common for unique items like antiques, Dutch auction is preferred for perishable items or items losing value over time (Li and Kuo, 2013). Primary advantages of the Dutch auction include its speed and transparency due to the fact that there are not more bids than items being sold for a single item (Fine, 2008). Moreover, the Dutch auction has an advantage over other auction types since it can be modified slightly by running the auction until a second price is accepted by buyers. This setting is similar to second-price auctions. Also, the setting can be adapted for a multiple-item case of descending price auction (Mishra and Parkes, 2009). Moreover, the Dutch auction clock method is very efficient for small lot trading in a fixed time and fixed number of transactions. Due to the clock speeds and small lots, the effect on cognitive capabilities of buyers favors the higher prices for auctioneers and gives them a revenue maximization opportunity (Ajit and Eric van, 1998). Examples of Dutch auctions include fish markets (Gallegati et al., 2011), firm slot sales for road transporters (Keating et al., 2008), flower auctions (Ajit and Eric van, 1998) and initial public offerings (IPO) (Biais and Faugeron-Crouzet, 2002).

3.2.2 Descending Price (Dutch) Auction as a Tolling Mechanism

There are several similarities between a revenue-maximizing toll authority and the Dutch auction markets, which lead to the idea of adapting the Dutch auction as an option for toll road operations.

The first similarity is that the tolling market is also a one-sided market as in the case of flower or fish markets. In this kind of market, an auctioneer cries out the prices and the process; the sale is then binding for the seller as seller cannot withdraw the offer without the buyer rejecting it. On the other hand, buyers decide whether to accept a bid or not. Once a buyer commits to the offer and the auction ends, the buyer cannot refuse the transaction (Mishra and Garg, 2006). The toll authority setting can be considered as a similar mechanism; the toll authority is responsible for setting the toll prices during a time period and drivers decide whether to pay the price or not. Once a driver commits to booking a slot on the toll road, he/she cannot refuse to pay and instead use the general lane.

In fish markets (or similar perishable goods markets), using the Dutch auction mechanism provides the ability to capture the highest price a buyer is willing to pay before the product perishes. In such markets, suppliers experience a time pressure since product expiration leads to financial loss. Similarly for the toll authority, the capacity of a toll lane is only available within a specific time interval. If the toll operator misses the chance to sell the slots beforehand, it results in forgone revenue for the tolling agency. Therefore, the reservation of slots can be treated as a perishable item. Also, as stated earlier, the Dutch auction is effective in maximizing the revenue from non-unique perishable items (Li and Kuo, 2011).

The third similarity is related to the time pressure on the buyers/clients in the auction. In the allotted time, the clients must assess the utility gain between waiting for a lower price and accepting the higher price in order to win and end the auction. For example, in the fish market, the buyers have a dilemma of buying the fish at a high price for the end user, or waiting for lower prices with risk of losing the auction, and not being able get the fish. In the same manner, for drivers deciding whether to go on toll road or not, for the sake of saving time and not experiencing congestion, the running toll clock creates a fear of losing the auction and pressure to accept higher prices.

For the listed reasons above, the approach documented here considers a Dutch auction mechanism of multiple identical items with unit demand implemented in a toll plaza scenario (Mishra and Garg, 2006). Multiple unit algorithms can be used with either uniform price formation or a discriminatory pricing mechanism in which bidders pay amounts dependent on the number of bids (Shen and Su, 2007). In the proposed methodology, the prices are announced on an in-vehicle screen at predetermined time intervals starting from the highest possible price. The price continues to fall until reaching a predetermined reserve price along the decision corridor. The drivers are aware of that they are being offered the same price as the drivers travelling within their proximity, or in other words, with drivers who have spent a similar amount of time negotiating over the toll price.

Our case with multiple buyers and multiple identical items for sale can be considered a one to one matching between driver and toll road slot. Multiple item Dutch auction have been studied by Mishra and Garg (2006). Uniform Dutch auction pricing is already investigated as a “Smart Market for Passenger Road Transport (SMPRT)” algorithm implemented in London for congestion pricing (Markose et al., 2007). Instead, we focus on discriminatory Dutch auction pricing. To accomplish this, we propose a change to the algorithm that represents important departure from the theoretical approach defining Dutch auctions. This leads to a similarity between the first-price auctions and the implemented algorithm in this study. In the proposed auction algorithm, the decrementing prices are announced individually to each driver eligible for bidding. Later, dependent on the capacity available, the winning vehicles are determined based on how high the price accepted and the priority level calculated from discrete bid levels are. This approach is useful due to revenue-maximizing behavior of toll authorities while ensuring that the drivers are provided with reliable travel time and a high level of service.

3.2.3 Key Features of the Proposed Tolling Mechanism

While the toll operator tries to maximize the total revenue, drivers are heterogeneous and like to minimize their travel times and costs. The Dutch auction mechanism implemented within the simulation incorporates these essential behavioral aspects of the toll operator and the drivers. The main features of the model developed can be described as follows:

- The use of the Dutch auction helps to exploit the highest possible value that a driver is willing to pay for booking a slot on the toll road. This can be considered as the basis for maximizing the revenue for toll operators.
- The auction approach also exploits the differences in drivers’ willingness to pay, which stem from different socio-economic backgrounds. Therefore drivers’ individual prices differ from each other based on several factors as explained in detail in the following sections.
- Finally, drivers would like to minimize their travel times along the corridor. How much they value the minimization of travel time and whether they are willing to pay for the savings depends on the given traffic conditions they experience and their socio-economic status. In the mechanism implemented, drivers make choices about whether or not to use the toll road.

3.3 OVERVIEW OF THE MODELING METHODOLOGY

In this section, the simulation model built in VISSIM 6.0 Microscopic Software is explained in detail. First, the hypothetical layout, which is considered as the initial test bed for the proposed algorithm, is introduced. The considered model layout built in VISSIM is combined with an external controller script written in VBA Programming Language which implements the descending price auction mechanism and desired driver behavior. The ABM portion of the model is composed of components. The parts, explained below, are agents of the model, an implemented bidding mechanism based on a logit choice model with a specific utility and probability function, and feedback control mechanism for capacity calculation and travel time savings.

3.3.1 Network Layout

In this study, a tolling scenario on a simple highway layout is considered to explore the effects of a Dutch auction mechanism on revenue generated from a hypothetical tolling system. The social welfare of the drivers in terms of average travel time savings between toll and general purpose lanes is also of interest. In the hypothetical layout, all drivers are assumed to have the same origin and destination for their trips with a toll plaza in between as shown in Figure 5 below. It is assumed that both toll lane and general lane have equal distance and capacity. In this layout, drivers are assumed to first travel over the decision corridor (main road) where they accept or reject the announced bid levels. Over the decision corridor, drivers either choose to stay in general purpose lanes or continue onto the toll road by paying the toll price determined by the auction.

Figure 5: Hypothetical transportation network diagram

In the simulation model, it is assumed that the main corridor has four lanes which further split into two separate roads: two toll lanes and two general purpose lanes. In order to create travel time differences between the two facilities, the two lanes on each facility reduce to a single-lane road.

3.3.2 Basic Tolling Mechanism and Agents

There are two types of decision-maker agents in the proposed toll algorithm: the drivers and the toll operator. They are making decisions interactively through V2I technologies based on the real time traffic conditions collected during the simulation and based on the personal preferences and differences.

In order to understand the roles of agents in the model and the basics of the tolling mechanism, consider a single driver-agent and toll-operator-agent. As the driver-agent enters the main corridor, he/she is presented with the highest possible price as the first toll offer. The driver-agent evaluates this broadcasted price offer for the toll lane. If his/her utility gain associated with the toll road over the general purpose road exceeds the driver-agent's acceptance threshold value, then driver-agent sends an acceptance message to the toll-operator. Otherwise, he/she sends a rejection message. When the driver-agent rejects the price, the toll-operator-agent decreases the offer by a predetermined decrement rate in the next round of bidding cycle and increments his/her bid levels by 1. The driver-agent is offered decreasing prices until a certain number of discrete bid levels are reached. The last toll price offered at the last (maximum) bid level corresponds to the reserve toll (price). When the driver-agent reaches that point by rejecting all previous offers, he/she will not be offered any lower toll rate, and has to travel on the general purpose lanes.

When there are numerous driver-agents in the system, then capacity of the toll road has to be considered. The toll-operator agent needs to check whether there is enough capacity for all driver-agents with accepted offers. After comparing the demand and capacity, toll-operator agent decides who to accept and who to reject starting from the highest price.

3.3.2.1 Drivers

In the model, driver-agents differ in several aspects to provide the necessary heterogeneity for ABMS. These aspects are differences in income levels and differences in value of time (VOT). Driver agents can also be categorized into different levels based on their willingness or satisfaction threshold to accept an arbitrary toll price offer and the utility gain at that price level. However, for the sake of simplicity, this threshold is considered as the same for all the driver agents in the network. The aforementioned differences for agent heterogeneity eventually affect the driver-agents' route choice behaviors, which are described with a logit model that will be explained later. Moreover, it is assumed that driver-agents know the estimated travel times on both toll and general purpose lanes in near real-time.

In the simulation model, the driver agent is responsible for the following decision-making processes:

- Evaluating the tolls broadcast by the toll-operator agent while considering the travel time savings (TTS) and his/her income level.
- Sending prompt messages whether to accept an offer or not.
- Changing lanes before reaching the junction for toll lanes if they are accepted to the toll lanes.

3.3.2.2 Toll-Operator

The other agent in the system is the toll-operator. The toll-operator agent is responsible for the following operations:

- Setting the highest possible bid and the reserve toll price.
- Announcing the driver-agent-specific prices based on the number of discrete bid levels a driver has been asked before.
- Checking the available capacity for booking slots at a certain time window for arrivals to the toll lane.
- Making decision on whether to accept a certain driver-agent at a certain price category to the toll road.
- Sending prompt messages to driver-agents whether their request to go on toll road is accepted or not

3.3.3 Income and VOT Distribution

One of the aims in this study is to discover the effects of socio-economic factors of drivers on route choice behavior. Therefore, the price discrimination of tolls based on several variables is implemented. The variables also create desired heterogeneity among driver-agents and a more realistic system. Therefore, factors such as gender, education level, and income level can be taken into consideration when modeling the drivers' route choice behavior based on the passengers' VOT in a price discrimination setting.

In the current version of the model, for the sake of simplicity, only difference in income levels factor into the calculation of VOT. According to this approach, driver-agents are segmented into income groups; the ones in the same category are assumed to have the same VOT distribution. The information on the driver-agent population and the income level categorization is adopted from a previous study (Cheng, 2013). Table 1 below shows the income segmentation.

Table 1: Segmentation of Driver Agents based on Income Levels

Driver's Income Level	% in driver population	Mean hourly income
High Income (>= 80k/year)	10	$49.8
Mid Income (>= 40k/year - <80k/year)	24	$28.8
Low Income (< 40k/year)	66	$9.6

It is also assumed that travel time valuation is taken as a percentage of the income per unit of time. In other words, driver-agents' VOT is a percentage of their mean hourly income. According to an NCHRP report, the percentage should be considered as 90% of the mean hourly income (Weisbrod, 2001). On the other hand, Livshits (2011) advise choosing a value between 50% and 70% . In this study, NCHRP Report's suggestion is adopted when calculating the VOT for each driver-agent segment as shown in Table 2 below.

Table 2: VOT Segmentation Based on Income Level

Driver's Income Level	VOT Category	VOT per hour
High Income (>= 80k/year)	High value of time	$44.82
Mid Income (>= 40k/year - <80k/year)	Mid value of time	$25.92
Low Income (< 40k/year)	Low value of time	$8.64

3.3.4 Drivers' Route Choice Behavior

Based on the income groups and different VOT categories above, a logit choice model was developed to describe the route choice behavior of the driver-agents. The driver-agents choose a route depending on the outcome of the logit function. The probability of choosing the toll road over the general purpose road based on the utility gain for a vehicle i at a time t is described as:

$$P(i,t) = \frac{1}{1 + e^{-\Delta U(i,t)}} \tag{16}$$

where $\Delta \mathrm{U}(\mathrm{i},\mathrm{t})$ is the utility gain or loss:

$$\Delta U\ (i,t) =\ U_t(i,t) -\ U_g(i,t) \tag{17}$$

$$U_t(i,t) = -\propto \times\ p(i,t) - \theta(i,t) \times\ TT_t \tag{18}$$

$$U_g(i,t) = -\theta(i,t) \times\ TT_g \tag{19}$$

where,

$U_t(i,t)$: the utility ofusing toll road at time t with price offer $\mathrm{p}(i,t)$, for vehicle i
$U_t(i,t)$: the utility ofusing general lane
$\propto$: change in utility per unit change in toll rate (cost coefficient)
$p(i,t)$: toll price on toll rate at time t for a particular driver i
$\theta(i,t)$: change in utility per unit change in travel time savings for driver i (time coefficient)
TT_t: travel time on toll road
TT_g: travel time on general road

Here, since there is no charge for general purpose lanes, the part associated with toll rate can be omitted from the calculation of $U_g(i,t)$. Therefore, the travel time savings affect the utility gain as:

$$\Delta U\ (i,t) =\ -\propto \times\ p(i,t) + \theta(i,t) \times (TT_g -\ \ TT_t) \tag{20}$$

In the route choice model described above, due to the differences in income levels and VOT, $\propto$ and θ coefficients are specific to each driver agent. In the proposed simulation model, θ is taken to be the same as the VOT values shown above in Table 2 (Cheng, 2013), $\propto$ is considered to be 1 as it is suggested in several studies (Livshits, 2011). It should be noted that the toll rate, $p(i,t)$, is specific to each driver-agent. This time-dependent toll rate is from the Dutch bidding mechanism and is a function of the number of past bid levels and the decrement rate as explained below.

3.3.5 Bidding Mechanism

As mentioned earlier, descending price auctions are one-sided auctions, in which auctioneer calls out the prices. In our model, the auctioneer is the toll-operator agent, who announces the highest bid and consecutive decrements in the prices to the driver-agents individually through vehicle-to-infrastructure (V2I) technology until a previously determined reserve price. Driver-agents, on the other hand, are buyers that decide whether to go on the toll road or not.

Before modeling the bidding mechanism, several assumptions are made:

- Driver-agents have the knowledge that prices are descending over the decision corridor; however, they do not have knowledge of whether there will be capacity available on the toll road; therefore they do not know if their request to enter the toll road can be accepted or not.
- Driver-agents also do not have knowledge of the reserve price, which marks the end of a bidding cycle for each driver-agent, or the next decremented price they might get as an offer from the toll-operator agent.

- On the other hand, the toll-operator agent has knowledge of drivers' reactions to each potential toll price and sets the highest acceptable bid and reserve price accordingly in order to maximize its revenue.

The combination of these conditions creates a dynamic environment. Each driver on the decision corridor faces a dilemma between rejecting a high price versus waiting for a lower price and potentially losing the auction due to capacity shortage. Each driver-agent needs to determine whether they gain more utility (value) by rejecting a high price offered by the toll-operator agent. Moreover, it is assumed that the driver-agents have knowledge of the estimated travel times at a near real-time. Thus, they estimate the travel time savings on the toll road.

Based on these assumptions, the mechanism works as follows:

Step 1 – Announcement of the individual prices by the toll authority

At regular time intervals, the toll-operator agent broadcasts individual toll prices to each eligible driver-agent. The driver-agents who have been offered but rejected the reserve price in the previous bidding cycle, if there was one, are no longer eligible for bidding since they reach the maximum number of discrete bid levels. Therefore, they are diverted to the general-purpose lanes.

Step 2 –Decision of drivers to take the toll road

Driver-agents evaluate the current traffic situation and decide whether they are better off entering the toll road at the current price offer. Their decision is dependent on their income level and VOT. If the calculated probability of choosing the toll lane over the general lane is over the driver-agent's acceptance threshold probability, he/she sends a prompt message through V2I channels to the toll authority agent requesting entry to the toll lane.

Step 3 – Capacity vs. demand check

The toll-operator agent, after collecting all of the confirmed bids, checks the projected available capacity on toll road and compares it with the demand. The demand over a certain time period in the future is calculated based on the vehicles' projected arrival times at the toll road entrance. Since toll-operator agent aims to provide a high level of service, he/she does not accept new driver-agents if the capacity is reached.

Step 4 – Sending acceptance/rejection messages to drivers

After checking the capacity and demand, the toll-operator agent determines the vehicles to be assigned to the toll road. If the number of confirmed bids is lower than the number of available projected capacity, the toll-operator agent accepts all the vehicles and sends them a prompt confirmation message. If the available capacity is lower than the number of vehicles willing to enter the toll road, then the toll–operator agent sorts the driver-agents' requests in a descending manner based on price and the number of previous bid levels. After sorting the drivers, the toll-operator-agent accepts vehicles one by one starting from the highest priority until all the available slots are sold. The prioritization of the vehicles and its details are explained in the following subsections.

3.3.5.1 *Calculation of maximum discrete bid levels and bid price offers*

Along the decision corridor, the toll-operator agent prompts the driver-agents for bids multiple times, which might cause distraction for driver-agents. In order to prevent distraction, the maximum possible number of bid announcements should be kept under a certain value, described as maximum number of discrete bid levels. It is calculated based on the distance of the decision area, time interval for toll price update, and free flow conditions. The value found is multiplied by 85% as shown in equation below in order to give driver-agents time for the necessary lane changes along the corridor and time delays in communication throughout the bidding process.

$$\beta_{max} = \left\lfloor {}^{D_{decision}}\!\big/_{(FFS \times \Delta t_\beta)} \times 0.85 \right\rfloor \tag{21}$$

where,

β_{max}: Maximum number of bid levels along the decision corridor
$D_{decision}$: The distance of the decision link
FFS: Free flow speed
Δt_β: Time interval between two bid announcement

After determining the maximum number of bid levels, the average price decrement between two consecutive bid announcements made by the toll operator is computed the following way:

$$\Delta p = \frac{p_{max} - p_{reserve}}{\beta_{max}} \tag{22}$$

where,

Δp: Price decrement rate
p_{max}: Maximum price for toll lane
$p_{reserve}$: Reserve price for toll lane

The prices that are offered between highest bid and reserve price are calculated based on the price decrement rate and number of previous discrete bids as shown below:

$$p(i,t) = p_{max} - \Delta p \times n(i,t) \tag{23}$$

where,

$p(i,t)$: Price offer to driver i at time t
$n(i,t)$: The number of discrete bid levels driver i has been offered

3.3.5.2 *Sorting and prioritizing drivers for acceptance to toll road*

There are two important variables to check when calculating the priorities of driver-agents for acceptance to the toll road. The first is how high the accepted toll price is for each driver-agent. The second variable is at what discrete bid level the driver-agent accepts the toll price offer. Therefore, first, driver-agents are sorted in a descending order according to the tolls they accepted.

If there is a tie, then whoever has the highest number of previous bid levels has the priority. If a tie occurs in this variable too, the driver-agents are sorted arbitrarily. After prioritization, driver-agents with the highest priority are accepted to toll road until the available capacity is used up.

3.3.6 VISSIM Model and VBA Script

In this section, the details of simulation model built in VISSIM and controlled through the VBA script are given. The reason that the external VBA controller was built is that in VISSIM, there is no way to access to each vehicle and add individual parameters required by the proposed methodology. Vehicles can be accessible only externally, and a separate class formed to assign and keep personal preference data can be stored.

The hypothetical layout built in VISSIM is shown below. In the simulation model, in order to collect the real time data and implement the results as a feedback control to the system, three travel time data collection measurements are built along with data collection points in every mile. These measurement points collect data for average speed limit, average flow, and travel time along the corridor.

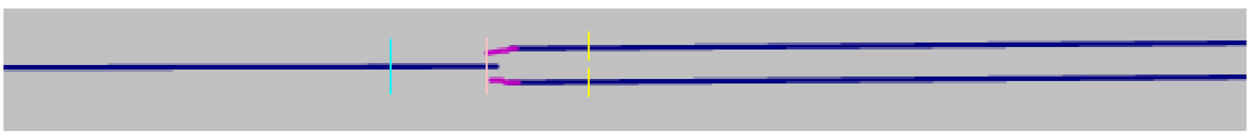

Figure 6: Hypothetical test bed layout on VISSIM

3.3.6.1 Model Inputs

In the simulation model, there are important variables related to bidding price offers, speed limits, desired capacity/flow mechanism, and the time interval for the toll clock. These variables play an important role in the initialization of the model. For example, desired speed limit on the toll lanes and general purpose lanes can be adjusted according to the model user's preferences. The change in speed limit will affect the suggested hourly flow rate (HCM (2000)) necessary to provide a high level of service on toll lanes.

3.3.6.2 Model Outputs

The simulation model allows recording a rich set of outputs and performance measures. The model is configured to record the following performance measures.

- Number of cars willing to go on toll lane at time period t
- Number of cars accepted to toll lane at time t
- Average toll price paid at each time interval
- Average number of discrete bid levels
- The distribution of drivers accepted to toll road by income segments
- Total revenue per time interval

3.3.6.3 *Flow Diagram of Main Functions in the Script*

The main data flow diagram of the proposed algorithm is shown in Figure 7 below. For further information on the other flow diagrams describing the components of the script, the reader is referred to Appendix B.

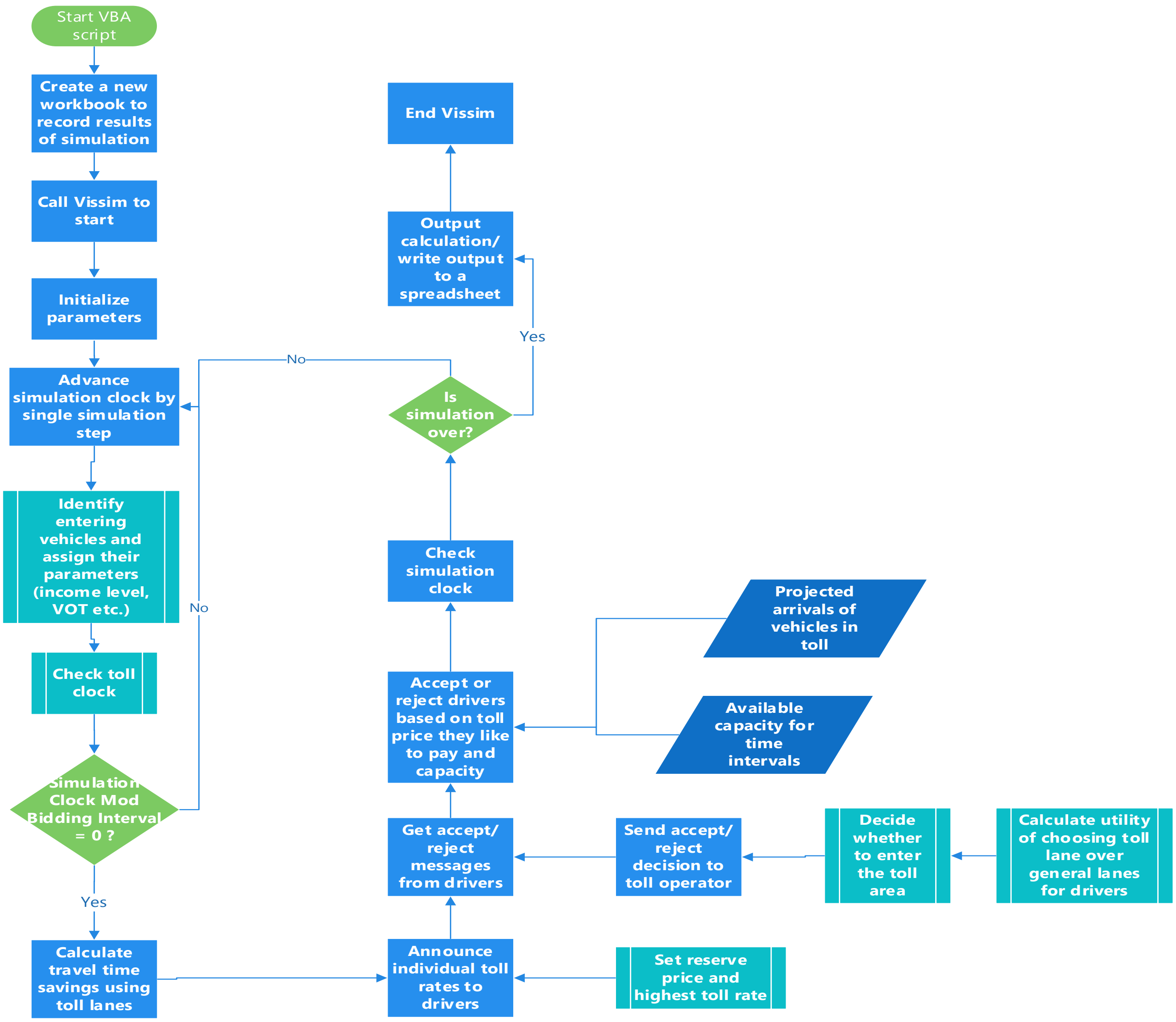

Figure 7: Main Flow Diagram of the Proposed Script and Bidding Mechanism

3.3.7 Current Status and Future Simulation Work

Currently, the VISSIM simulation model and its VBA scripts are being finalized. Initial testing indicates that the behavioral elements are coded correctly. Numerical experiments will be designed and conducted to evaluate how the system will perform under the Dutch action described above. In these experiments, the traffic congestion, VOT and its distribution, and road capacity will be varied to understand the impacts of these variables on the system performance and total toll revenue. The simulation model will be also used in Phase II of the project to test alternative action mechanisms for toll road operations.

4.0 PHASE II OF THE RESEARCH

In the research discussed in this report, the project team developed the analytical solutions for a new tolling approach based on a combinatorial Vickrey auction designed for a single toll road with multiple entry points. This the proposed scenario, travelers can make multiple bids to gain access to part or the entire toll lane. The impacts of varying the distribution of travelers' Value of Time (VOT) on the revenue earned by the toll operator have been analyzed by making simplifying assumptions about the behavior of users.

In Phase II of the study, the team proposes developing and conducting surveys to gain insights into how people would choose to travel on toll roads when they are given the opportunity to bid. Surveys provide a means to collect some information on individuals' bidding behavior, even if only stated preferences, and can be used to form the foundation of the human behavior in our model. Modeling human behavior is challenging, especially when accounting for heterogeneous behavior of drivers. Recently, a new approach for modelling human behaviors within agent-based model (ABM) was published by the leading ABM expert in the world, Joshua Epstein (2014). The approach is called Agent_zero and it overcomes some of the existing problems of human modeling within an ABM environment, e.g., limitations of adaptive behaviors. Thus, the focus of Phase II of this research is collection of survey data of stated preference of individual behavior within a future tolling scenario that requires V2I communication; analysis and incorporation of survey data results into existing auction model; and simulation of a new auction model using the Agent_zero approach.

The main outcome of this next phase will be the survey data, its analysis, and the results from implementing socio-behavioral data into an agent-based simulation of a connected vehicle environment. The results of this research will be presented at professional conferences (e.g., TRB) and published in archived journals.

For the next phase of the project, the team will be joined by Dr. Lei Zhang. Dr. Zhang from the University of Maryland and is Associate Professor and Director of the National Transportation Center in the Department of Civil and Environmental Engineering at the University of Maryland, College Park. Dr. Zhang has published more than 150 peer-reviewed journal and conference papers on topics including transportation planning, transportation economics and policy, travel behavior, advanced travel demand modeling, transportation data and survey methods, and traffic operations. His expertise on congestion pricing, agent-based modeling, and simulation are particularly relevant to the Phase II of the project.

5.0 PROJECT TEAM

The Principal Investigator of this project is Dr. Mecit Cetin who has expertise in congestion pricing and tolls and published several journal papers in this area. Dr. Cetin is an associate professor in the Department of Civil and Environmental Engineering and the Director of the Transportation Research Institute (TRI) at ODU. Dr. Cetin has more than 11 years of experience in the areas of transportation modeling, congestion pricing, and intelligent transportation systems. He has published 25 journal articles and more than 40 papers in peer-reviewed international conference proceedings. He is a member of TRB's Urban Transportation Data and Information Systems Committee (ABJ30) and Artificial Intelligence and Advanced Computing Applications Committee (ABJ70). He is a guest editor for the Journal of Intelligent Transportation Systems for a special issue on Cyber Transportation Systems and Connected Vehicle Research.

Also from ODU, Drs. Mike Robinson and Andrew Collins will be co-principal investigators on this project. Dr. Mike Robinson is an Assistant Research Professor at the ODU Virginia Modeling, Analysis, and Simulation Center (VMASC) and the Director of ODU's Center for Innovative Transportation Solutions. Dr. Robinson is a member of the TRB Emergency Evacuation Task Force, The Transportation Cyber Security Subcommittee, and the Pedestrian Modeling and Simulation Subcommittee.

Dr. Collins is also an Assistant Research Professor at VMASC and has expertise in agent-based modeling and game theory. Dr. Andrew Collins' Ph.D. research was on the application of reinforcement methods to dynamic pricing games in the airline industry and has applied such techniques to Agent-based Simulation. Dr. Collins is currently working on the impact of traffic incidents on evacuations times and the impact of foreclosure on property markets.

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APPENDIX A

NUMERICAL RESULTS

Table A-1: Numerical results from optimization problem with VOT following a triangular distribution with mean of 0.5 and variance of 1/24

Capacity				Acceptance Percentage			
OB(toll)	OB(GP)	BD(toll)	BD(GP)	A(OD)	A(OB)	A(BD)	Return
1	1	1	1	7.60%	9.50%	9.50%	0.0181
0.5	1	0.5	1	7.00%	8.75%	8.75%	0.0176
1	0.5	1	0.5	7.65%	9.57%	9.57%	0.2910
0.5	0.5	0.5	0.5	7.65%	9.57%	9.57%	0.2905
0.5	0.5	1	1	7.65%	9.57%	9.57%	0.1544
1	1	0.5	0.5	7.65%	9.57%	9.57%	0.1544
1	1	1	0.5	7.65%	9.46%	9.58%	0.1546
1	0.5	1	1	7.65%	9.58%	9.46%	0.1546
0.5	1	1	1	7.29%	8.50%	9.77%	0.0180
1	1	0.5	1	7.29%	9.77%	8.50%	0.0180

Table A-2: Numerical results from optimization problem with VOT following a log normal distribution with mean of 0.5 and variance of 1/24

Capacity				Acceptance Percentage			
OB(toll)	OB(GP)	BD(toll)	BD(GP)	A(OD)	A(OB)	A(BD)	Return
1	1	1	1	5.99%	10.00%	10.00%	0.0178
0.5	1	0.5	1	5.57%	9.33%	9.33%	0.0175
1	0.5	1	0.5	6.03%	10.06%	10.06%	0.2857
0.5	0.5	0.5	0.5	5.99%	10.00%	10.00%	0.2853
0.5	0.5	1	1	5.99%	10.00%	10.00%	0.1516
1	1	0.5	0.5	5.99%	10.00%	10.00%	0.1516
1	1	1	0.5	6.02%	9.98%	10.06%	0.1517
1	0.5	1	1	6.02%	10.06%	9.98%	0.1517
0.5	1	1	1	5.77%	9.17%	10.17%	0.0177
1	1	0.5	1	5.77%	10.17%	9.17%	0.0177

Table A-3: Numerical results from optimization problem with VOT following a beta distribution with mean of 0.5 and variance of 1/24

Capacity				Acceptance Percentage			
OB(toll)	OB(GP)	BD(toll)	BD(GP)	A(OD)	A(OB)	A(BD)	Return
1	1	1	1	7.74%	9.56%	9.56%	0.0182
0.5	1	0.5	1	7.10%	8.80%	8.80%	0.0178
1	0.5	1	0.5	7.80%	9.64%	9.64%	0.2915
0.5	0.5	0.5	0.5	7.74%	9.56%	9.56%	0.2910
0.5	0.5	1	1	7.74%	9.56%	9.56%	0.1546
1	1	0.5	0.5	7.74%	9.56%	9.56%	0.1546
1	1	1	0.5	7.80%	9.51%	9.64%	0.1549
1	0.5	1	1	7.80%	9.64%	9.51%	0.1549
0.5	1	1	1	7.41%	8.52%	9.85%	0.0180
1	1	0.5	1	7.41%	9.85%	8.52%	0.0180

Table A-4: Comparisons of three VOT distributions

Capacity				Distribution Return			Compared to Triangular Distribution	
OB(toll)	OB(GP)	BD(toll)	BD(GP)	Triangular	Log Normal	Beta	Log Normal	Beta
1	1	1	1	0.0181	0.0178	0.0182	-1.66%	0.55%
0.5	1	0.5	1	0.0176	0.0175	0.0178	-0.57%	1.14%
1	0.5	1	0.5	0.291	0.2857	0.2915	-1.82%	0.17%
0.5	0.5	0.5	0.5	0.2905	0.2853	0.291	-1.79%	0.17%
0.5	0.5	1	1	0.1544	0.1516	0.1546	-1.81%	0.13%
1	1	0.5	0.5	0.1544	0.1516	0.1546	-1.81%	0.13%
1	1	1	0.5	0.1546	0.1517	0.1549	-1.88%	0.17%
1	0.5	1	1	0.1546	0.1517	0.1549	-1.88%	0.17%
0.5	1	1	1	0.01796	0.0177	0.0180	-1.45%	0.09%
1	1	0.5	1	0.01796	0.0177	0.0180	-1.45%	0.09%

APPENDIX B

SIMULATION MODEL FLOW DIAGRAMS

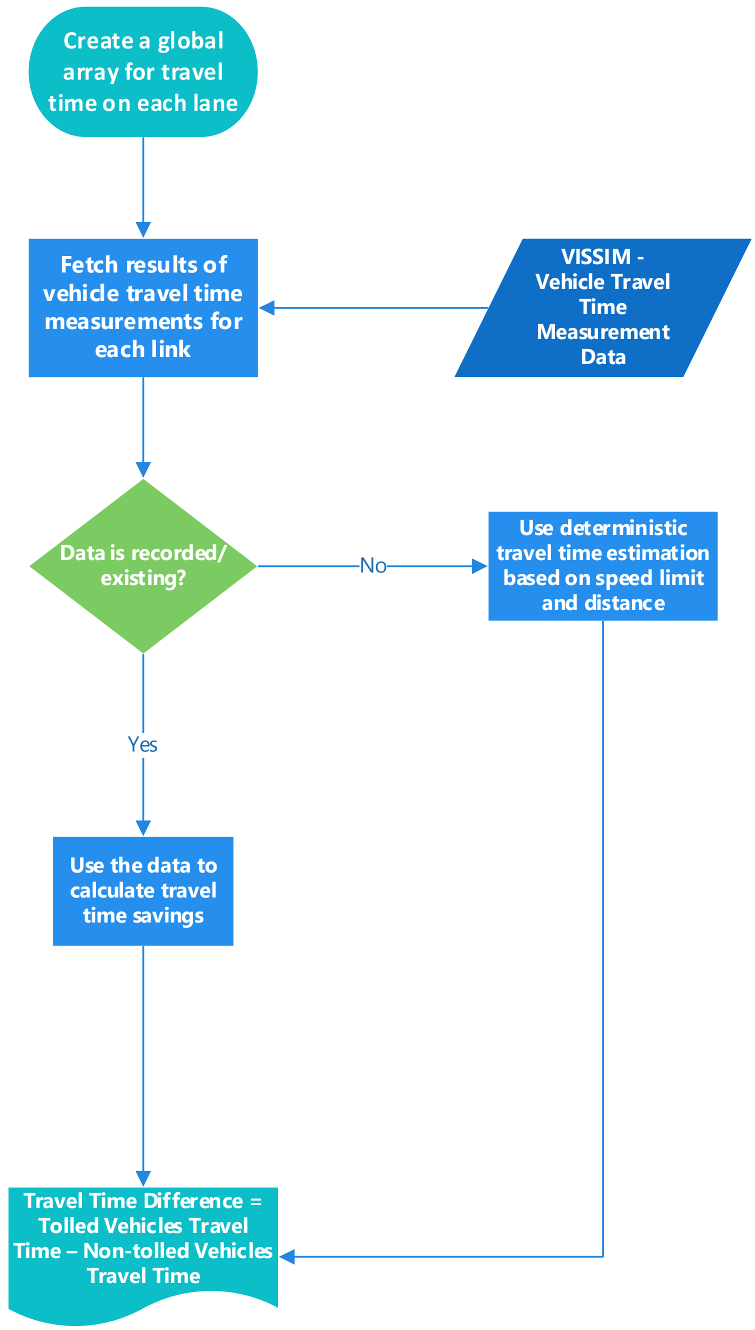

Figure B-1: Calculation of travel time savings within simulation

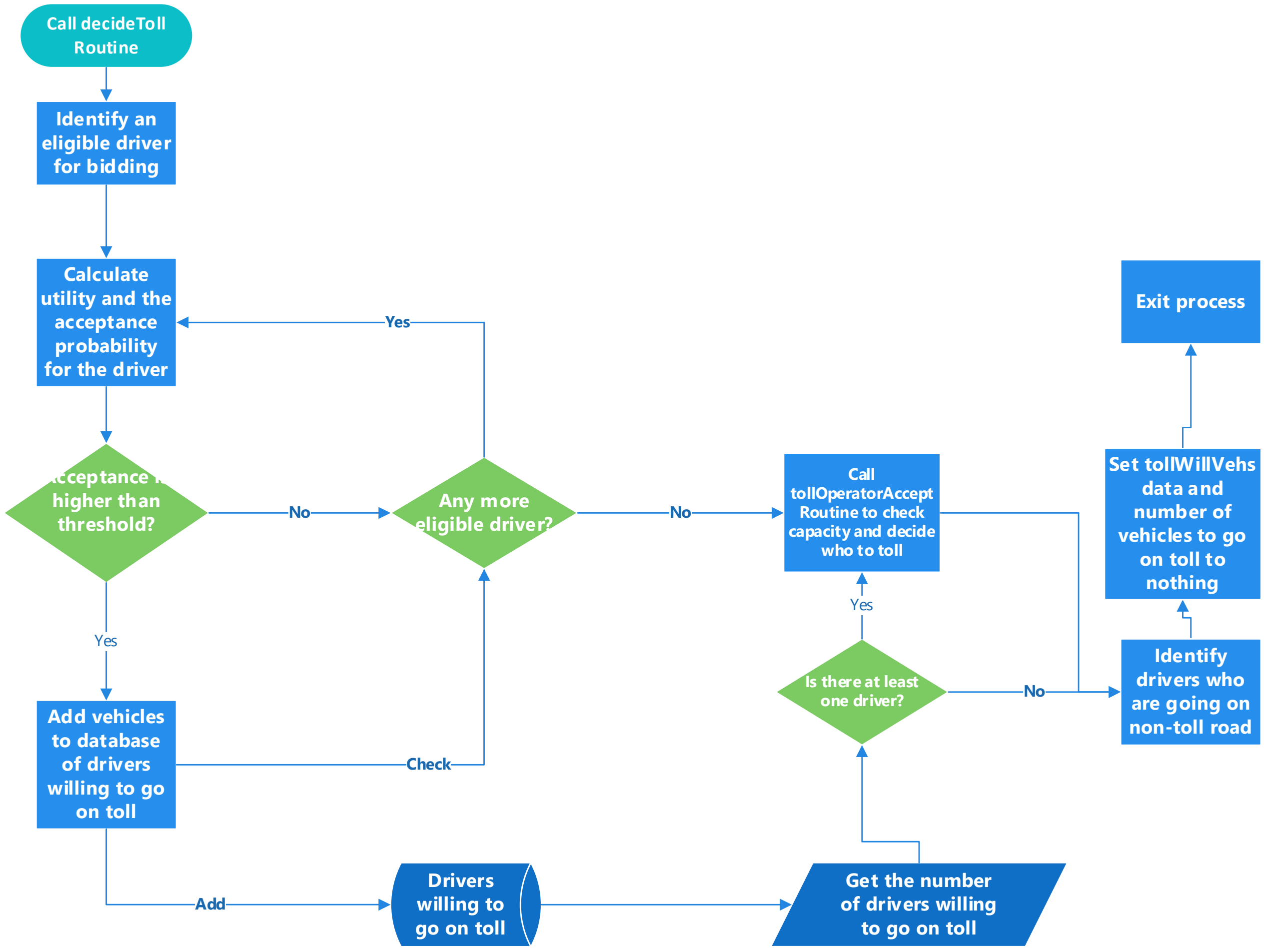

Figure B-2: Decision making process of accepting or rejecting a present price of toll plazas

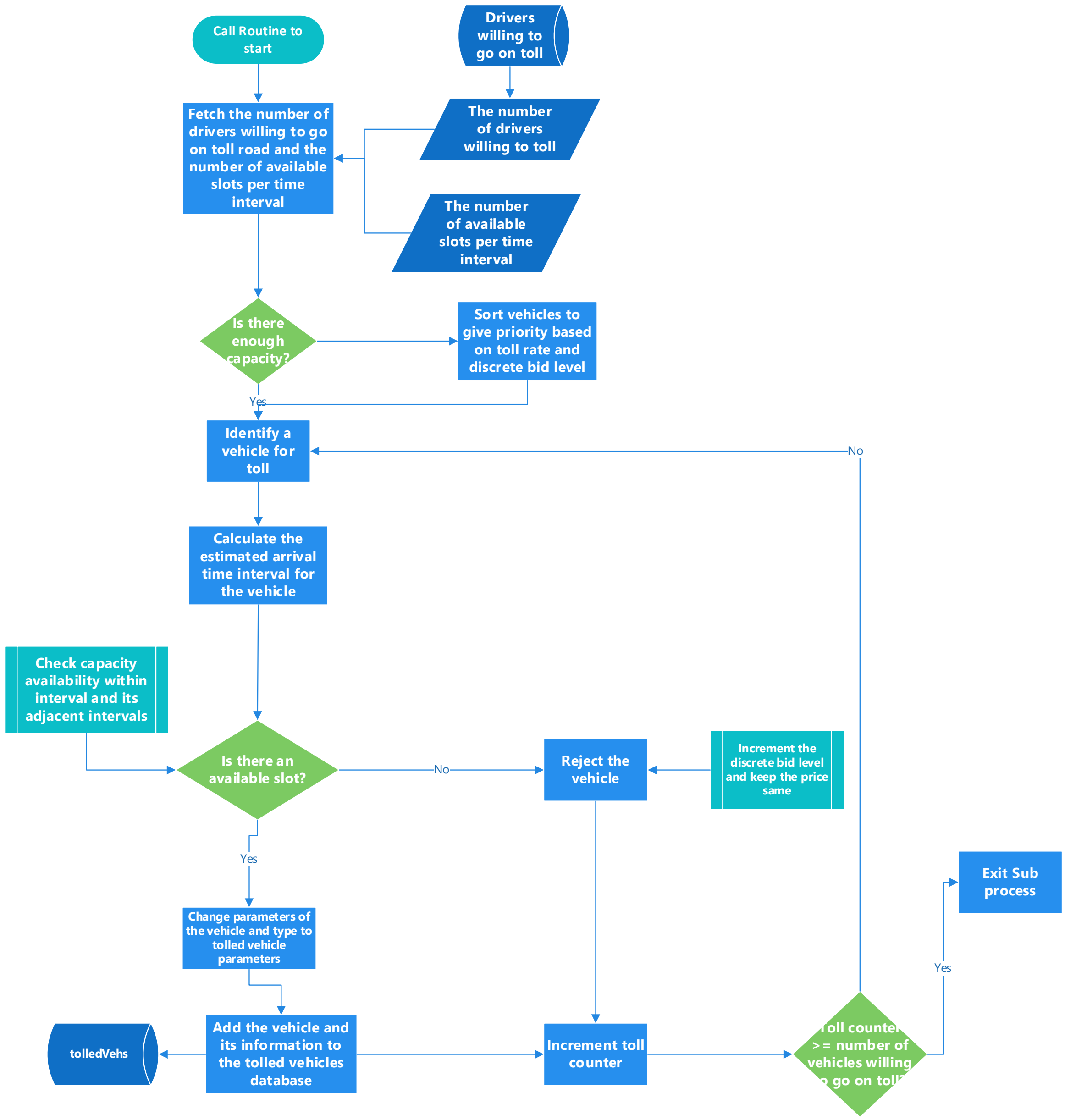

Figure B-3: Toll authority assessment – Acceptance or rejection based on capacity